Tutorial on Quantum Annealing

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You have a jupyter notebook (access here) at your disposal to help you to solve some of the following exercises. This example code solves the trivial Diophantine problem $x + y = 6$.

Problem 1: Warm-up

Note: for this problem you will not need the example code.

Reduce the following Hamiltonian

$$H = \tau_1\tau_2\tau_3 - \tau_2\tau_3\tau_4 + \tau_1\tau_2\tau_3\tau_4 \,. \tag{1}$$

- Which is the most convenient way to reduce it minimising the number of auxiliary qubits?
- Use whatever you want (pen and paper, Python, Mathematica, etc.) to write the expression of the quadratised Hamiltonian. Check that the position and the degeneracy of the minimum is preserved.
- *Homework*: How would you imagine a general algorithm to reduce a generic higher-order Hamiltonian to a quadratic one using the minimum number of auxiliary qubits? Write your own code in your preferred programming language to apply the reduction algorithm. You can randomly generate an Hamiltonian and collect the couplings in a set of arrays. For example $[1, 2, 3, -6]$ would represent a cubic coupling between qubit 1, 2 and 3 with strength -6. Apply then the algorithm manipulating the arrays.

Problem 2: Taxicab numbers

Adapt the code to find the first few taxicab numbers

$$a^3 + b^3 = c^3 + d^3 \,, \tag{2}$$

where a, b, c, d are all different. You will need to find a way to encode the constraint $a \neq c, d$ (or similar for b) in the annealer.

Problem 3: Continuous optimisation problems

Adapt the code to minimise the following function

$$F(x) \;=\; x^4 - \frac{23}{2}x^3 + \frac{753}{16}x^2 - \frac{161}{2}x\,. \tag{3}$$

Note: one of the solutions is not an integer. You will need a slightly different encoding for the x variable.